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method of determining truth. The highest authority in matters religious ought to be the truth, and we deem it our duty to search for the truth with the most rigorous method that science places at our disposal.

P. C.

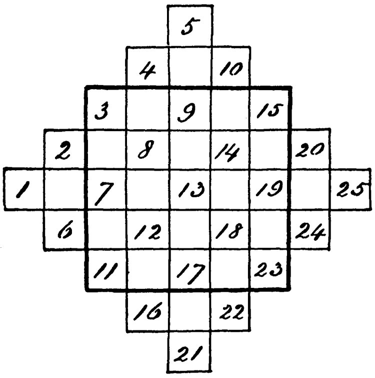
REFLECTIONS ON MAGIC SQUARES.

MATHEMATICAL, HISTORICAL, AND PHILOSOPHICAL.

Mathematics, especially in the field where it touches philosophy, has always been my foible, and so Mr. W. S. Andrews' article on "Magic Squares" tempted me to seek a graphic key to the interrelation among their figures which should reveal at a glance the mystery of their construction.

THE ORDER OF FIGURES.

In magic squares of odd numbers, 3 × 3, 5 × 5, 7 × 7, etc., there is no difficulty whatever, as Mr. Andrews' diagrams show at a glance. The consecutive figures run up slantingly in the form of a staircase, so as to let the next higher figure pass over into the next higher or lower cell of the next row, and those figures that according to this method would fall outside of the square, revert into it as if the magic square were for the time (at the moment of crossing its boundary) connected with its opposite side into the shape of a cylinder. This cannot be done at once with both its two opposite vertical and its two opposite horizontal sides, but the pro-



3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

A SPECIMEN OF 5 × 5 MAGIC SQUARE.

cess is easily represented in the plane by having the magic square extended on all its sides, and on passing its limits on one side we must treat the extension as if we had entered into the magic square on the side opposite to where we left it. If we now transfer the figures to their respective places of the inside square, they are shoved over in a way which by a regular transposition will counteract their regular increase of counting and so equalize the sums of entire rows.

The case is somewhat more complicated with even magic squares, and a suggestion which I propose to offer here, pertains to their formation. Mr. Andrews begins their discussion by stating that "in perfect magic squares of this class it is necessary that the sum of each column shall be the same amount, and also that the sum of any two numbers that are geometrically equidistant from the center of the square shall equal the sum of the first and last numbers of the series."

The smallest magic square of even numbers is, of course, 4 × 4; and he points out that if we write the figures in their regular order in a 4 × 4 square, those standing on the diagonal lines can remain in their places, while the rest are to be reversed so as to replace every figure by its complementary to 17 (i. e., 2 by 15, 3 by 14, 5 by 12, 9 by 8) the number 17 being the sum of the highest and lowest numbers of the magic square (i. e., $n^2 + 1$). It is by this reversal of figures that the inequalities of the natural order are equalized again, so as to make the sum of each row equal to 34, which is one fourth of the sum total of all figures, the general formula being:

$$\frac{1+2+3+4+\ldots . n}{n} = \frac{n}{2}(n^2+1).$$

We will now try to find out more about the relation which the magic square arrangement bears to the normal sequence of figures.

For each corner there are two ways, one horizontal and one vertical, in which figures can be written in the normal sequence; accordingly there are altogether eight possible arrangements, from which we select one as fundamental, and regard all others as mere variations, produced by inverting and reversing the order.

As the fundamental arrangement we choose the ordinary way of writing from the left to the right, proceeding in parallel lines downward. We call this "the ordinary order" or *o*. Its reverse proceeds from the lower right hand corner toward the left, and line by line upward, thus beginning the series where the ordinary arrangement ends, and ending where it started. We call this order "the reversed ordinary," or simply *ro.*

Another order is produced by following the Hebrew and Arabic mode of writing: we begin in the upper right-hand corner, proceeding to the left, and then continue in the same way line by line downward. This, the inverse direction to the ordinary way, we call briefly *i.*

The reverse order of *i,* starting in the lower left corner, proceeding to the right, and then line by line upward we call *ri.* Further on we shall have occasion to represent these four orders by the following symbols: *o* by ●; *ro* by ⊛; *i* by ✠; *ri* by +.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

ORDER *o* (●).

6	5	4	3	2	1
12	11	10	9	8	7
18	17	16	15	14	13
24	23	22	21	20	19
30	29	28	27	26	25
36	35	34	33	32	31

ORDER *i* (✠)

31	32	33	34	35	36
25	26	27	28	29	30
19	20	21	22	23	24
13	14	15	16	17	18
7	8	9	10	11	12
1	2	3	4	5	6

ORDER *ri* (+).

36	35	34	33	32	31
30	29	28	27	26	25
24	23	22	21	20	19
18	17	16	15	14	13
12	11	10	9	8	7
6	5	4	3	2	1

ORDER *ro* (⊛).

It will be noticed that *i* is the vertical mirror picture of *o* and *ro* of *ri,* and *vice versa.* Further if the mirror is placed upon one of the horizontal lines, *ri* is the mirror picture of *o* as well as *ro* of *i* and *vice versa.*

There are four more arrangements. There is the Chinese way of writing downward in vertical columns as well as its inversion, and the reversed order of both. This method originated by the use

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

1	143	142	4	5	139	138	8	9	135	134	12
132	14	15	129	128	18	19	125	124	22	23	121
120	26	27	117	116	30	31	113	112	34	35	109
37	107	106	40	41	103	102	44	45	99	96	48
49	95	94	52	53	91	90	56	57	87	86	60
84	62	63	81	80	66	67	77	76	70	71	73
72	74	75	69	68	78	79	65	64	82	83	61
85	59	58	88	89	55	54	92	93	51	50	96
97	47	46	100	101	43	42	104	105	39	38	108
36	110	111	33	32	114	115	29	28	118	119	25
24	122	123	21	20	126	127	17	16	130	131	13
133	11	10	136	137	7	6	140	141	3	2	144

EVEN SQUARES IN MULTIPLES OF FOUR.*

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* These squares, 4 × 4 and its multiples, consist of *o* and *ro* orders only, and it will be sufficient to write out the two 4 × 4 squares, which show how *o* and *ro* are mutually interchangeable.

o	*ro*	*ro*	*o*
ro	*o*	*o*	*ro*
ro	*o*	*o*	*ro*
o	*ro*	*ro*	*o*

ro	*o*	*o*	*ro*
o	*ro*	*ro*	*o*
o	*ro*	*ro*	*o*
ro	*o*	*o*	*ro*

of bamboo strips as writing material in China, and we may utilize the two vowel sounds of the word "bamboo" (viz., *a* and *u*) to name the left and the right downward order, *a* the left and *u* the right, the reverse of the right *ru* and of the left *ra*; but for our present purpose there will be no occasion to use them.

Now we must bear in mind that magic squares originate from the ordinary and normal consecutive arrangement by such transpositions as will counteract the regular increase of value in the normally progressive series of figures; and these transpositions depend upon the location of the several cells. All transpositions in the cells of even magic squares are brought about by the substitution of figures of the *ro, i,* and *ri* order for the original figures of the ordinary or *o* order, and the symmetry which dominates these changes becomes apparent in the diagrams, which present at a glance the order to which each cell in a magic square belongs.

Numbers of the same order are grouped not unlike the Chladni acoustic figures, and it seems to me that the origin of the regularity of both the magic figures and this phenomenon in acoustics, is due to an analogous law of symmetry.

The dominance of one order *o, ro, i,* or *ri,* in each cell of an even magic square, is simply due to a definite method of their selection from the four different orders of counting. Never can a figure appear in a cell where it does not belong by right of some regular order, either *o, ro, i,* or *ri.*

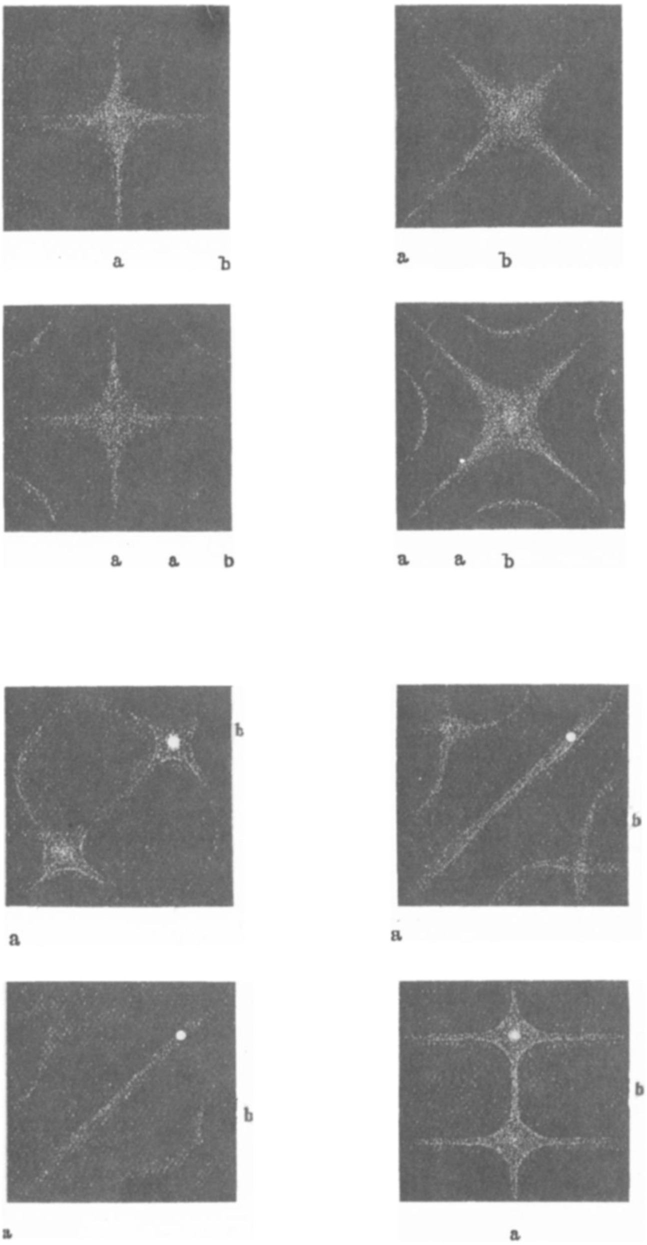
1	35	4	33	32	6
12	8	28	27	11	25
24	17	15	16	20	19
13	23	21	22	14	18
30	26	9	10	29	7
31	2	34	3	5	36

1	5	33	34	32	6
30	8	28	9	11	25
18	23	15	16	20	19
24	14	21	22	17	13
7	26	10	27	29	12
31	35	4	3	2	36

6 × 6 EVEN SQUARES.

The magic square of 4 × 4, consists only of *o* and *ro* figures, and the same rule applies to the simplest construction of even squares of multiples of four, such as 8 × 8, and 12 × 12.

There are several ways of constructing a magic square of 6 × 6. Our first sample consists of 12 *o,* 12 *ro,* 6 *ri,* and 6 *i* figures The 12 *o* hold the diagonal lines. The 12 *ro* go parallel with one of



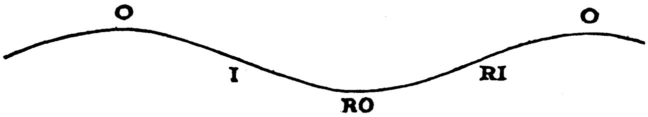
CHLADNI FIGURES.[*]

[*] The letter *a* indicates where the surface is touched with a finger; while *b* marks the place where the bow strikes the glass plate. In the four upper

these diagonals, and stand in such positions that if the whole magic square were diagonally turned upon itself, they would exactly cover the 6 *i,* and 6 *ri* figures. And again the 6 *i* and 6 *ri* also hold toward each other places in the same way corresponding to one another; if the magic square were turned upon itself around the other diagonal, each *ri* figure would cover one of the *i* order.

If we compare the magic squares with the sand-covered glass plates which Chladni used and think of every cell as equally filled with the four figures that would fall upon it according to the normal sequence of *o, ro, i,* and *ri*; and further if we compare their change into a magic square to a musical note harmonizing whole rows into equal sums, we would find (if by some magic process the different values of the several figures would mechanically be turned up so as to be evenly balanced in rows) that they would present geometrically harmonious designs as much as the Chladni acoustic figures.

The progressive transformations of *o, ro, i,* and *ri,* by mirroring, are not unlike the air waves of notes in which *o* represents the crest of the wave, *ro* the trough, *i* and *ri* the nodes.



In placing the mirror at right angles progressively from *o* to *i,* from *i* to *ro,* from *ro* to *ri,* and from *ri* to *o,* we return to the beginning thus completing a whole sweep of the circle.* The reverse of *o* which is *ro* represents one-half turn, *i* and *ri* the first and third quarter in the whole circuit, and it is natural, therefore, that a symmetry-producing wave should produce a similar effect in the magic square to that of a note upon the sand of a Chladni glass plate.

MAGIC SQUARES IN SYMBOLS.

The diagrams which are offered here, as samples, are the best evidence of their resemblance to the Chladni figures, both exhibiting in their formation, the effect of the law of symmetry. The most elegant way of rendering the different orders, *i, ri, o,* and *ro,* visible at a glance, would be by printing the cells in four different colors,

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diagrams the plate has been fastened in the center, while in the lower ones it has been held tight in an excentric position, indicated by the white dot.

* See diagram on page 125.

but for proving our case, it will be sufficient to have the four orders represented by four symbols, omitting their figure values, and we here propose to indicate the order of *o* by ●, *ro* by ❁, *i* by ✠, *ri* by +.

●❁❁●●❁❁●
❁●●❁❁●●❁
❁●●❁❁●●❁
●❁❁●●❁❁●
●❁❁●●❁❁●
❁●●❁❁●●❁
❁●●❁❁●●❁
●❁❁●●❁❁●

8 × 8. 32 *o* and 32 *ro*.

●❁❁●●❁❁●●❁❁●
❁●●❁❁●●❁❁●●❁
❁●●❁❁●●❁❁●●❁
●❁❁●●❁❁●●❁❁●
●❁❁●●❁❁●●❁❁●
❁●●❁❁●●❁❁●●❁
❁●●❁❁●●❁❁●●❁
●❁❁●●❁❁●●❁❁●
●❁❁●●❁❁●●❁❁●
❁●●❁❁●●❁❁●●❁
❁●●❁❁●●❁❁●●❁
●❁❁●●❁❁●●❁❁●

10 × 10. 72 *o* and 72 *ro*.

SQUARES OF MULTIPLES OF FOUR.
Constructed only of *o* and *ro*.

●✠❁❁❁❁✠●
+●✠❁❁✠●+
❁+●●●●+❁
❁❁●●●●❁❁
❁❁●●●●❁❁
❁+●●●●+❁
+●✠❁❁✠●+
●✠❁❁❁❁✠●

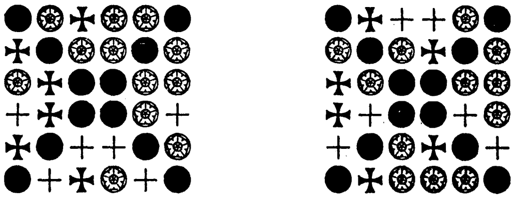
●❁+●●+❁●
●❁+●●+❁●
✠❁●❁❁●❁✠
●+❁●●❁+●
●+❁●●❁+●
✠❁●❁❁●❁✠
●❁+●●+❁●
●❁+●●+❁●

8 × 8 SQUARES.
Constructed from all the orders, *o, ro, i,* and *ri.*

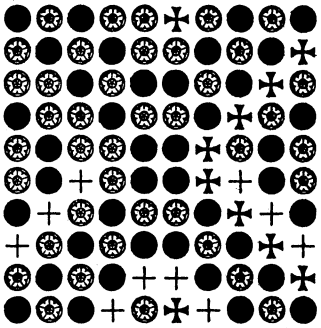
●✠++++✠●
✠●❁❁❁❁●✠
❁❁●✠✠●❁❁
++✠●●✠++
++✠●●✠++
❁❁●✠✠●❁❁
✠●❁❁❁❁●✠
●✠++++✠●

ANOTHER 8 × 8 SQUARE.

It will be noted that in this square the arrangement of the *o* symbols corresponds very closely to the distribution of the sand in the second of the Chladni diagrams. The same may be said of the two following figures, and it is especially true of the first one of the 8 × 8 squares just preceding.



12 *o,* 12 *ro,* 6 *i,* 6 *ri.*



40 *o,* 40 *ro,* 10 *i,* 10 *ri.*

The reader will notice that there is a remarkable resemblance between the symmetry displayed in this figure and in the fourth of the Chladni diagrams.

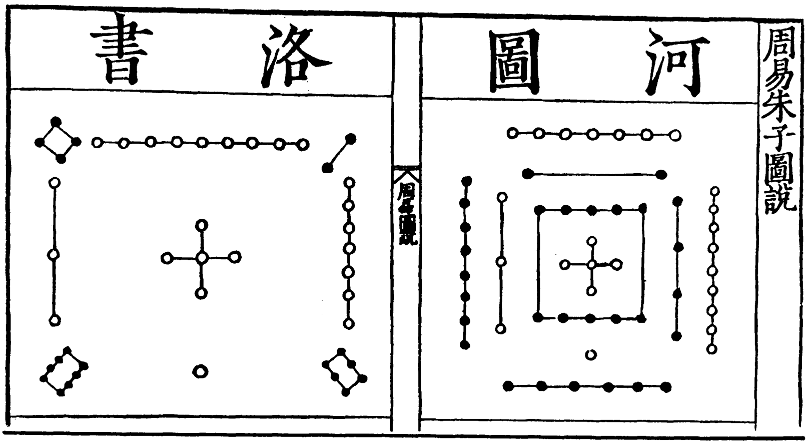
EXAMPLES OF 6 × 6 AND 10 × 10 MAGIC SQUARES.

THE MAGIC SQUARE IN CHINA.

In the introduction to the Chou edition of the *Yih King,* we find some arithmetical diagrams and among them the *Loh-Shu,* the scroll of the river Loh, which is a mathematical square from 1 to 9, so written that all the odd numbers are expressed by white dots, i .e., yang symbols, the emblem of heaven, while the even numbers are in black dots, i. e., yin symbols, the emblem of earth. The invention of the scroll is attributed to Fuh-Hi, the mythical founder of Chinese civilization, who according to Chinese reports lived 2858-2738 B. C. But it goes without saying that we have to deal here with a reconstruction of an ancient document, and not with the document itself.

The first unequivocal apearance of the Loh-Shu in the form of a magic square is in the latter part of the posterior Chou dynasty (951-1126 A. D.) or the beginning of the Southern Sung dynasty (1127-1333 A. D.). The Loh-Shu is incorporated in the writings of Ts'ai Yüan-Ting who lived from 1135-1198 A. D. (cf. Mayers, *Chinese Reader's Manual,* I, 754a), but similar arithmetical diagrams are traceable as reconstructions of primitive documents among scholars that lived under the reign of Sung Hwei-Tsung, which lasted from 1101-1125 A. D. (See Mayers, *C. R. M.,* p. 57.)

The *Yih King* is unquestionably very ancient and the symbols yang and yin as emblems of heaven and earth are inseparable from its contents. They existed at the time of Confucius (551-479 B. C.), for he wrote several chapters which are called appendices to the *Yih King,* and in them he says (III, I, IX, 49-50. *S. B. E.,* XVI, p. 365.):



THE SCROLL OF LOH. THE MAP OF HO.*
(According to Ts'ai Yüang-tĭng.)
TWO ARITHMETICAL DESIGNS OF ANCIENT CHINA.

"To heaven belongs 1; to earth, 2; to heaven, 3; to earth, 4; to heaven, 5; to earth, 6; to heaven, 7; to earth, 8; to heaven, 9; to earth, 10.

"The numbers belonging to heaven are five, and those belonging to earth are five. The numbers of these two series correspond to each other, and each one has another that may be considered its mate. The heavenly numbers amount to 25, and the earthly to 30. The numbers of heaven and earth together amount to 55. It is by these that the changes and transformations are effected, and the spiritlike agencies kept in movement."

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* The map of Ho properly does not belong here, but we let it stand because it helps to illustrate the spirit of the times when the scroll of Loh was composed in China. The map of Ho contains five groups of odd and even figures, the numbers of heaven and earth respectively. If the former are regarded as positive and the latter as negative, the difference of each group will uniformly yield + 5 or — 5.

This passage was written about 500 B. C. and is approximately simultaneous with the philosophy of Pythagoras in the Occident, who declares number to be the essence of all things.

One thing is sure, that the magic square among the Chinese cannot have been derived from Europe. It is highly probable, however, that both countries received suggestions and a general impulse from India and perhaps ultimately from Babylonia. But the development of the yang and yin symbols in their numerical and occult significance can be traced back in China to a hoary antiquity so as to render it typically Chinese, and thus it seems strange that the same idea of the odd numbers as belonging to heaven and the even ones to earth appears in ancient Greece.

I owe the following communication to a personal letter from Professor David Eugene Smith of the Teachers' College of New York:

"There is a Latin aphorism, probably as old as Pythagoras, *Deus imparibus numeris gaudet.* Virgil paraphrases this as follows: *Numero deus impare gaudet.* (Ecl. viii, 75). In the edition I have at hand* there is a footnote which gives the ancient idea of the nature of odd and even numbers, saying:

"*...impar numerus immortalis, quia dividi integer non potest, par numerus mortalis, quia dividi potest; licet Varro dicat Pythagoreos putare imparem numerum habere finem, parem esse infinitum* [a curious idea which I have not seen elsewhere]; *ideo medendi causa multarumque rerum impares numeros servari: nam, ut supra dictum est, superi dii impari, inferi pari gaudent.*

"There are several references among the later commentators to the fact that the odd numbers are masculine, divine, heavenly, while the even one were feminine, mortal, earthly, but I cannot just at this writing place my hands upon them.

"As to the magic square, Professor Fujisawa, at the International Congress of Mathematicians at Paris in 1900, made the assertion that the mathematics derived at an early time from the Chinese (independent of their own native mathematics which was of a somewhat more scientific character), included the study of these squares, going as far as the first 400 numbers. He did not, however, give the dates of these contributions, if indeed they are known."

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* P. Virgilii Maronis | Opera, | cum integris commentariis | Servii, Philargyrii, Pierii, | Accedunt | Scaligeri et Lindenbrogii | . . . . . | Pancratius Masvicius | . . . | Tom. I, | . . . | Leonardiae, | . . . | . . cIↄIↄccxvii.|

As to other magic squares, Professor Smith writes in another letter:

"The magic square is found in a work by Abraham ben Ezra in the eleventh century. It is also found in Arabic works of the twelfth century. In 1904, Professor Schilling contributed to the Mathematical Society of Göttingen the fact that Professor Kielhorn had found a Jaina inscription of the twelfth or thirteenth century in the city of Khajuraho, India, a magic square of the notable peculiarity that each sub-square sums to 34."

Here is the square which Professor Smith encloses:

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

We must assume that we are confronted in many cases with an independent parallel development, but it appears that suggestions must have gone out over the whole world in most primitive times perhaps from Mesopotamia, the cradle of Babylonian civilization, or later from India, the center of a most brilliant development of scientific and religious thought.

How old the magic square in China may be, is difficult to say. It seems more than probable that its first appearance in the twelfth century is not the time of its invention, but rather the date of a recapitulation of former accomplishments, the exact date of which can no longer be determined.

THE JAINA SQUARE.

Prof. Kielhorn's Jaina square is not a "perfect magic square" according to Mr. Andrews' definition, quoted above. While the sums of all the rows, horizontal, vertical, and diagonal, are equal, the figures equidistant from the center are not equal to $n^2 + 1$, viz., the sum of the first and last numbers of the series. Yet it will be seen that in other respects this square is more perfect, for it represents a distribution of the figure values in what might be called absolute equilibrium.

First we must observe that the Jaina square is continuous, by which I mean that it may vertically as well as horizontally be turned upon itself and the rule still holds good that wherever we may start four consecutive numbers in whatever direction, backward or forward, upward or downward, in horizontal, vertical, or slanting lines, always yield the same sum, viz. 34, which is $2(n^2+1)$; and so does any small square of 2×2 cells. Since we can not bend the square upon itself at once in two directions, we make the result visible in the adjoined diagram, by extending the square in each direction by half its own size.

10	5	16	3	10	5	16	3
15	4	9	6	15	4	9	6
1	14	7	12	1	14	7	12
8	11	2	13	8	11	2	13
10	5	16	3	10	5	16	3
15	4	9	6	15	4	9	6
1	14	7	12	1	14	7	12
8	11	2	13	8	11	2	13

THE SQUARE EXTENDED.

Wherever 4×4 cells are taken out from this extended square, we shall find them satisfying all the conditions of this peculiar kind of magic squares.

The construction of this ancient Jaina equilibrium-square requires another method than we have suggested for Mr. Andrews' "perfect squares," and the following considerations will afford us the key:

First we write the numbers down into the cells of the square in their consecutive order and call the four rows in one direction A, B, C, D; in the other direction 1, 2, 3, 4. Our aim is to redistribute them so as to have no two numbers of the same denomination in the same row. In other words, each row must contain one and only one of each of the four letters, and also one and only one of each of the four figures.

	1	2	3	4
A	1	2	3	4
B	5	6	7	8
C	9	10	11	12
D	13	14	15	16

In Consecutive Order.

A_1	B_4	C	D
D_2	C_3	B	A
3	2		
4	1		

The Start for a Redistribution.

A_1	B_4	C_2	D_3
D_2	C_3	B_1	A_4
B_3	A_2	D_4	C_1
C_4	D_1	A_3	B_2

The Perfected Redistribution.

1	8	10	15
14	11	5	4
7	2	16	9
12	13	3	6

Figure Values of the Square.

CONSTRUCTING THE SQUARE.

We start in the left upper corner and write down in the first horizontal row the letters A, B, C, and D, in their ordinary succession, and in the second horizontal row, the same letters in their inverted order. We do the same with the numbers in the first and second vertical rows. All that remains to be done is to fill out the rest in such a way as not to repeat either a letter or a number. In the first row there are still missing for C and D the numbers 2 and 3, of which 2 must belong to C, for C_3 appears already in the second row and 3 is left for D.

In the second row there are missing 1 and 4, of which 1 must belong to B, because we have B_4 in the first row.

In the first vertical row the letters B and C are missing, of which B must belong to 3, leaving C to 4.

In the second vertical row A and D are missing for 1 and 2. A_1 and D_2 exist, so A must go to 2, and D to 1.

In the same simple fashion all the columns are filled out, and then the cell names replaced by their figure values, which yields the same kind of magic square as the one communicated by Prof. Smith, with these differences only, that ours starts in the left corner with number 1 and the vertical rows are exchanged with the horizontal ones. It is scarcely necessary to point out the beautiful symmetry in the distribution of the figures which becomes fully apparent when we consider their cell names. Both the letters, A.

B, C, D, and the figures, 1, 2, 3, 4, are harmoniously distributed over the whole square, so as to leave to each small square its disdinct individuality, as appears from the following diagram:

A	B	C	D
D	C	B	A
B	A	D	C
C	D	A	B

1	4	2	3
2	3	1	4
3	2	4	1
4	1	3	2

THE ARRANGEMENT OF LETTERS AND FIGURES.

The center square in each case exhibits a cross relation, thus:

C	B
A	D

3	1
2	4

In a similar way each one of the four groups of four cells in each of the corners possesses an arrangement of its own which is symmetrically different from the others.

* * *

I have had some correspondence on the Jaina square with Mr. W. S. Andrews, and after a perusal of my article he sends me the following suggestions:

"There are some secondary features connected with magic squares apart from their chief characteristic that every main column sums up to the same total amount. Two of these secondary features are as follows:

"First—That any two numbers which are equidistant from the center of the square shall amount to the sum of the first and last numbers of the series.

"Second—That every sub-diagonal column shall sum up to the same total amount as the main columns, thus making a square which is 'continuous' in any direction from any column.

"The ordinary magic square of 4 × 4 possesses the first characteristic but not the second. The Jaina square possesses the second characteristic but not the first. The 5 × 5 magic square given in Fig. 5 of my article possesses both characteristics."

Mr. Andrews encloses the following diagrams, adding thereto a few explanations.

I.

A_1	D_3	D_2	A_4
C_4	B_2	B_3	C_1
B_4	C_2	C_3	B_1
D_1	A_3	A_2	D_4

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

ORDINARY 4 × 4 MAGIC SQUARE.

II.

B_3	C_4	A_1	D_2
A_2	D_1	B_4	C_3
D_4	A_3	C_2	B_1
C_1	B_2	D_3	A_4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

THE JAINA SQUARE.

III.

A_1	B_4	C_2	D_3
D_2	C_3	B_1	A_4
B_3	A_2	D_4	C_1
C_4	D_1	A_3	B_2

1	8	10	15
14	11	5	4
7	2	16	9
12	13	3	6

THE JAINA SQUARE MODIFIED BY DR. CARUS.

IV.

B_5	D_3	A_1	C_4	E_2
C_1	E_4	B_2	D_5	A_3
D_2	A_5	C_3	E_1	B_4
E_3	B_1	D_4	A_2	C_5
A_4	C_2	E_5	B_3	D_1

10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

5 × 5 MAGIC SQUARE.*

Referring to the small numbers in these squares it will be noticed that:

In square I,—The sum of any two numbers that are geometrically opposite to each other $= n + 1$.

In squares II and III,—The sum of the numbers in each column, *including the sub-diagonals,* is equal to $n(n+1)/2$.

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* See Fig. 5 of Mr. Andrews' article on "Magic Squares," in the October *Monist,* page 432.

This feature implies *continuity* in the derived magic squares.

In square IV,—*Both* of the above characteristics obtain.

Mr. Andrews adds that he has not succeeded in making any arrangement of letters and numbers in squares I, II, or III which will embody *both* of the above characteristics.

THE PHILOSOPHICAL SIGNIFICANCE OF NUMBER.

Although magic squares have no immediate practical use, they have always exercised a great influence upon thinking people. It seems to me that they contain a lesson of great value in being a palpable instance of the symmetry of mathmatics, throwing thereby a clear light upon the order that pervades the universe wherever we turn, in the infinitesimally small interrelations of atoms as well as in the immeasurable domain of the starry heavens, an order which, although of a different kind and still more intricate, is also traceable in the development of organized life, and even in the complex domain of human action.

Magic squares are a visible instance of the intrinsic harmony of the laws of number, and we are thrilled with joy at beholding this evidence which reflects the glorious symmetry of the cosmic order.

Pythagoras says that number is the origin of all things, and certainly the law of number is the key that unlocks the secrets of the universe. But the law of number possesses an immanent order, which is at first sight mystifying, but on a more intimate acquaintance we easily understand it to be intrinsically necessary; and this law of number explains the wondrous consistency of the laws of nature. Magic squares are conspicuous instances of the intrinsic harmony of number illustrating the harmony of the spheres, and so they will serve as an interpreter of the cosmic order that dominates all existence.

Magic squares are a mere intellectual play that illustrates the nature of mathematics, and. incidentally, the nature of existence dominated by mathematical regularity. They illustrate the intrinsic harmony of mathematics as well as the intrinsic harmony of the laws of the cosmos.

The wisdom which we can trace in the cosmic order is not artificial, not the product of premeditated thought, not the result of an excogitated plan, but innate in existence itself. It is necessary and absolute. It has not originated by the fiat of a God. It is without origin and without an end, being itself eternal and omnipresent; it is immutable, irrefragable, and unqualified, which means that it is without exception and its efficiency without conditions. It is uncaused, that is to say, it is not the effect of a cause, but is itself the principle of the law of causation.

Spinoza calls it "*causa sui*" (that is, cause of itself), but we take exception to this term as self-contradictory. There is no "cause of itself,"—that is, an effect that would be its own cause, nor is there an uncaused cause, a first cause. Cause and effect are always successive events; and every cause is the effect of a prior cause. Cause is some act or motion or disturbance of any kind, which, by a change in the system of conditions, calls forth further changes leading to a re-arrangement of the conditions, which is called the effect. What Spinoza means to say is that God or the absolute, which he calls "*substantia,*" is the universal *raison d'être* of all things, the final principle of explanation, and the supreme determinant.

Spinoza does not distinguish between two important ideas, *cause* and *reason,* and instead of speaking, as he intends to do, of the ultimate reason of the world, he speaks of the first cause, thus producing a confusion which down to the present day has proved a stumbling block to innumerable philosophers who succeeded him. Reason is the answer to the question *why.* Reason, unlike cause, which is always an event in the objective world, formulates a law that explains some event or class of events. Hence reason in the subjective sense can mean only our argument; and in the objective sense, the general law which dominates the course of events to be explained. All reasons intended to explain the isolated events of our experience, form a great system, and so comprehension consists practically in systematizing knowledge, and the different reasons can be subsumed under general laws, and can be represented as being finally reducible to one universal *raison d'être.*

All reasons considered by themselves, constructed independently of material existence, have been worked out in the sciences of pure form, (mathematics with all its branches, arithmetic, algebra, geometry, etc.; logic, and what Kant calls pure nature-science) which would comprise a critical and scientific view of causality, of the law of preservation of matter and energy, and kindred ideas.

And what is the reason that these so-called absolute branches of knowledge play such an important part in the comprehension of the world?

The purely formal sciences possess the strange quality of being constructed by the human mind without utilizing external experiences; they are what Kant calls "*a priori*," that is, they antedate experience; their validity is not based upon experience, but conditions it; they hold good before we begin to make experiences, and experience presupposes their validity. If they were not valid, experience would be impossible. While thus the formal sciences seem to be mind-made, they are after all the basic laws of objective existence, and the agreement between the basic laws of the mind and the highest laws of objective existence, does not teach us, as Kant says, that the order of the world is purely subjective, and transferred from the thinking mind upon the surrounding cosmos, from our own mental or ideal inside into the actual and real outside of space, from the subject to the object—a system which he calls "critical idealism"; it teaches us the intrinsic necessity of the universal laws governing any possible kind of existence; and we learn thereby that their basic principle may be condensed in the one word —consistency. It means that whatever originates, exists, or is created, (if there is any consistency at all) will according to conditions produce a definite kind of an order.

In arithmetic we create a universe of figures by the process of counting; in geometry we create another universe by drawing lines in the abstract field of imagination, laying down definite directions; in algebra we produce magnitudes of a still more abstract nature expressed by letters; and if we think of the material world as being created by a God, it must have been brought about by producing bodily forms. In all these cases the first step producing the general conditions in which we move, lays down the rule to which all further steps are subject, and so every one of these universes is dominated by a consistency, producing a wonderful symmetry, which in the cosmic world has been called by Pythagoras "the harmony of the spheres."

The intrinsic necessity of symmetry explains better than Kant's idealism why the mind-made laws of mathematics agree so closely with the laws of nature governing the material universe; both are applications of the same order underlying every consistent system, and we know very well that the mathematician creates, not in subjective arbitrariness, but after the intrinsic necessity of the first start from his position; and the objective realities, the laws of nature, which correspond to his theorems, are not nonentities, not abstractions without meaning, but they are the dominating factors that shape all things in particular and the world in general. The system of these factors has not been made by the creator to suit his plans of creation, but it is the worthiest part of the creator himself. Humanly speaking we may call it the eternal constitution of the creator's mind. It alone marks the divinity of God, whatever we may understand by that name, and it alone is the Godhead of God.

THE GOD PROBLEM.

I have discussed the God problem before, but the subject is of such great importance and far-reaching consequence, that I will discuss it again in this connection, even at the danger of repeating myself.

There are two hostile parties in religion confronting each other. The one explains the mystery of the world order by the assumption of a personal God, an omnipotent, divine personality, in whose image man has been created; a being, who, like man, is possessed of will, intelligence and sentiment, who thinks and acts and plans as we do; the other party reduces everything to chance with its infinite possibilities. The two theories exclude one another, so that if one be true the other is necessarily wrong. Advocates of the former believe that God fashioned the world order to suit his purpose; those of the latter look upon the universe as a soulless, mindless, meaningless congeries of matter, exhibiting in its whirl of commotion many accidental uniformities. It is frequently assumed that no third proposition is possible, since these two appear to be contradictory. Because the iconoclastic proposition is very unsatisfactory, theists argue that we should accept belief in a personal God, on the ground that it is after all the most comforting and satisfactory solution. We, however, deem both positions to be faulty, and their contradictory feature is due solely to a slight misunderstanding of the true nature of the constitution of the world, as well as the significance of the God idea. Thus we take a middle ground which concedes to both parties some of their most vital contentions, and at the same time criticizes both for misstatements, which necessarily obscure the whole issue.

On the one side, we grant to the atheist that the world order can not have been fashioned by a God, for it is itself immutable and eternal; but at the same time we criticise in the atheist's position his lack of appreciation of the world order as both the creative factor and the standard of moral conduct and truth; or, to speak religiously, as the divinity immanent in nature. We agree with the materialist that all concrete existence is material, we criticise him when he claims that there is no reality except that of matter. The interrelation of things, the formal aspect of being, and with them the laws of pure form, are not nonentities but the most essential features of reality. Neither matter nor energy is the most important part of the world. That which makes things such as they are is their form; and though the laws of form seem to be pure nothingness because they are not material entities, they are nevertheless the quintessence of being, for they alone, and not material things, condition the differences that constitute better and worse, higher and lower, valuable and worthless, good and evil, right and wrong, truth and falsehood. Stones and all other bodily objects are real; but the laws of nature, though not material, are after all actual presences in the world. They are more than real, they are superreal. Without them life would possess no significance and so we see that the materialist is onesided in his pretensions.

In extenuation of the errors of materialism we must remember that it was invented to deny the ghost existence of spirit. Spirit should not be regarded as a thing-in-itself. But neither should matter,—and as for that, spiritual life is as real as form, and the laws of form to which it owes its existence.

The theist appreciates the dignity of the cosmic order as a moral factor in the world, and is even right in representing it as the creator, the ruler, the supreme judge; but he is wrong in attributing to God human form, and the human constitution of individual existence. A personal God, (i. e., the deity conceived as an individual being) is a mere allegory, and the theist falls a prey to the pagan tendency of primitive man when he takes religious allegories in a literal sense and insists on the very points where the allegory fails.

While the method of representing God as a person is justifiable as an allegory, because the symbol itself is dignified and appropriate (man's personality is after all the highest type among all the creatures in the whole realm of creation), we must not forget that the very attributes of divinity exclude some of those features which are essential in the personality of man. The attributes of God are eternality, omnipresence, universality, omniscience, unqualified justice, spirituality, etc. Eternality excludes at once the assumption of God's thought being successive like the thinking of man. In

Isaiah (lv. 9) God says,—"As the heavens are higher than the earth, so are....my thoughts than your thoughts," and we insist that the difference betwen God's thought and man's is not even a mere difference of degree. While man's thoughts are temporal, being successive in time, God's are above time: they are simultaneous and omnipresent like the laws of nature. They are not dependent on time and space, but are operative in potent and immediate efficacy. Man's consciousness is the product of mental activity; it originates from feeling, and is practically nothing but an intensified state of sentiency. It is needed as a guide for man's life, so as to make it possible for him to adapt himself to circumstances. God is not in need of any adaptation to circumstances. He is not possessed of a consciousness like that of man, and the consistent conception of omniscience excludes the mechansim of consciousness which admits the several ideas, or objects of thought only successively within the limits of special attention. Omniscience means that all the divine thought is at once present in God. As eternality means that they are above time, so omniscience means that they are above space. They are not concentrated on a special spot after the fashion of cerebral activities in man, but they are independent of any mechanism. The materialist would say that because they are not material things such as stones or our own bodies, that they are non-existent; but we claim that for all that they are efficient; and if they are not real in the narrow sense of the word, if they are not concrete objects, they are nevertheless efficient presences, and so we rightly call them super-real.

The same is true as to the other divine attributes. The interadjustment of the whole harmony that pervades the cosmic order is so perfectly balanced that no fault can be found with it, and where we would take exception to the evils of existence, we will always find out, if we only patiently investigate, that the basic principle is right and could not be changed. Justice in the constitution of the world is nothing but the recognition of right adjustment, and so whatever the ills of life may be, God will remain justified.

While the iconoclasts, atheists, materialists, and kindred thinkers, look upon the uniformities of nature as accidental, we regard them as intrinsically necessary. According to our experience we find nothing in life which is not dominated by uniformity, and all events that seem to be isolated, and would not admit of being classified in the system of uniformities built up by science, are simply evidences of our ignorance, and though there be many of them, there is no scientist who would look upon them in a different light. The whole world with all its phases, is subject to this immutable order of natural laws, and the assumption of accidental origin is absolutely excluded. The world order is a reality, or rather a superreality. It is possessed of all the divine attributes, and being a systematic whole, it constitutes a specific organism of a definite character, comparable to a personality.

While the cosmic order is not like man, we recognize at once that man was created in its likeness; for man originated by a recognition of universals, by discovering the general in the particular. The precedence of man over the beast is his reason, and man's reason (the *logos* of Neo-Platonism) is nothing but the becoming conscious of the uniformities of nature and interpreting them as universals, as instances of a type, and by comprehending the laws in which single cases find their application.

This idea has been indicated in the Gospel where it is said that the logos (that is the logical world order) was with God in the beginning and that it was God himself. Man, however, has become man only by a participation in the logos. To the extent that the logos has become incarnate in man, man has realized his humanity, and the highest ideal of manhood is the God-man, the logos incarnate.

Though God is not personal in the sense of human personality, we must not say that he is *im*personal, for that would appear as if divinity were less than man. The very opposite is the case. God being the prototype of man, man being formed in the likeness of God by incorporating in himself the divine logos, we must recognize that God, the prototype of all personality, is not less than a person, but more than a person, and noticing that the totality of divine thought which we trace in the laws of nature, constitutes a system not unlike the organism of a personal being, we prefer to characterize God as superpersonal. The negation of the human in God does not deprive him of anything divine, and the divine features which make man akin to God, are better, nobler and more divine than they are in any individual personality.

We do not limit God to the laws of nature, although we do not hesitate to characterize our God-conception as "Nomotheism." But by *nomos* we understand any norm, and God is the entirety of all that is eternally and universally normative. This eternal norm shows itself most palpably in the laws of reason and laws of nature, but we do not mean to limit our view of God to that feature of the cosmic order which the naturalist formulates as natural laws, and the logicion as logical theorems. That aspect of God which is formulated by science is only a part of his being, and important though it may be, it is not the only important one. There are other aspects within the reach of our knowledge, and other additional possible ones. There is the aspect which the mystic takes in his devout attitude of vague feeling; there is the aspect of adoration in the worshiper; there is the aspect of aspiration to an ideal that glows in the soul of the moralist, and none of them is sufficient to exhaust the significance of the divine presence in which we live and move and have our being. No fault can be found with any attempt of the human mind to reach out in appreciation of the divine spirit, nor in any yearning for a closer and more intimate union with him, if only those who accept these several aspects remain conscious of their insufficiency, and would not, as does the dogmatist, take allegories literally and look upon their own individual and particular standpoint as the only possible and only allowable one.

I am far from saying that all aspects are of equal value: on the contrary, we can easily discriminate between the lower and the higher, between the more or less true, between the more or less matured, more or less free from superstition, more or less purified, more or less ennobling. I insist only on the insufficiency of any one of them which leaves out a recognition of the others.

God, although not a material being, although nowhere in space, and never in time, nowhere here or there, and never now and then, is not a nonentity. Though he is not a force, nor an energy, nor even the sum total of energy in the world, he is not inefficient. On the contrary he is omnipotent. His power is not to be measured in foot-pounds, yet his will, if we may use this humanizing term allegorically, is irresistible. He is an all-efficient presence in all happenings, and there is nothing that does not subsist through him.

Nomotheism is not pantheistic. Pantheism identifies God and the universe, and according to nomotheism, God is different from the concrete and material universe. God is in the universe, and it so happens that the old paradox is justified, that the part is greater than the whole. But God is not only present in the universe, he is present even in non-existence; that is to say the purely formal laws of being hold good not only for the actualized world, but for any possible world, for all the future, for all the past, for all the systems of thought constructed by science, and even for those universes that will never be created. The eternal norm of the divine principle is actualized in nature, but it is above nature, because nature is, as it were, only a single instance of the infinite possibilities to which it applies or could apply. Thus God is not limited to the cosmic order of nature, but is supernatural in the literal sense of the word. No system of things or beings, either real or imaginary, could exist without him, except perhaps a chaos; yet even that would be so filled with self-contradictory inconsistencies or irrational unthinkables that (if it be a true chaos) it would be thinkable only as a vague idea; it could not stand, for it would be doomed to die even before an attempt were made to call it into existence.

There is no prophet that preaches the superpersonal God more plainly than mathematics, and the magic squares are like a magic mirror which reflects a ray of the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos not less than in the mysterious depths of the human mind.

EDITOR.

MR. PETERSON'S PROPOSED DISCUSSION.

Very valuable ideas ofttimes appear so obvious, when once set forth, that high laudation of their inventors would invite ridicule. Such, we are told, was the notion that obseded C. Colombo, and such is Mr. Peterson's proposal to start in *The Monist* a discussion of philosophical terminology. It may be a very simple proposal, but nobody, as far as one careful reader of *The Monist* remembers, had made it before; and its utility to students of phenomenology, normative science, and metaphysics will have a high co-efficient in its proportionality to the advantage they take of it. Duty calls upon us to contribute, each one what he can that will be useful, whether in the way of question or in that of answer. It seems likely that in my life-time of study I may have learned something of the way to investigate questions such as Mr. Peterson puts; and if so, here is an opportunity to be of aid to other students.

Experience, the first term concerning which Mr. Peterson asks for light, is somewhat remarkable for having been employed as nearly as possible in the same sense from Polus the Acragentine (i. e. native of Girgenti) sophist down to Avenarius and Haeckel.